is doing when one asserts or endorses a theoretical claim, in turn, demands the acquisition of language and logoi and acquaintance with the relations between them and the dimension of structure. The cultivation of *sm* and *sf*-abilities, therefore, can be elaborated with reference to the fundamental aspects of theory—the mutual relations between language and logoi (*L*), structure (*S*) and the data under consideration (*U*) provided by *L*—within which objectivity is determined:

(A) ***sm*-abilities**: Generally, semantic abilities can be characterized as qualitative higher-level modes of cognition. They afford agent models that are qualitatively compressed and therefore economical. Such models are complex and dynamically stable yet small in size (semantics as a qualitative mode of compression). Semantic abilities can be roughly defined as absolutely necessary abilities for the structuration of the world, i.e., the function of the irreducible correspondence of mind-language in relating to the world. Therefore, semantic abilities are abilities that permit the state of affairs concerning *that which is* to be rendered intelligible, thought, or spoken of. Primarily, ontological facts are configurations of semantic facts.[216] *sm*-abilities mainly involve *conceptualization*. In Brandomian terms semantic abilities can be approximately characterized as those abilities-or-practices necessary or sufficient to obtain semantic relations between vocabularies and those abilities-or-practices necessary or sufficient for deploying vocabularies that stand in semantic relations to one another. Put differently, semantic abilities concern what one must do so as to count as saying something *meaningful*, judging something, or thinking about various kinds of things, and what one must say in

216 'The ontological structures emerge directly from the semantic ones in that [...] semantics and ontology are two sides of the same coin. The fundamental ontological "category" (according to traditional terminology) is the "primary fact"; all "things" (in philosophical terms, all "beings" or "entities") are configurations of primary facts. The term "fact" is taken in a comprehensive sense, corresponding to the way this term is normally used at present (e.g., "semantic fact", "logical fact," etc.). It therefore does not necessarily connote, as it does in ordinary terminology, the perspective of empiricism.' Puntel, *Structure and Being*, 15.

order to explicitly specify or codify practices underlying those sayings or thinkings. Here and throughout this book, 'meaning' stands only for determinate semantic value as that which is assigned to a piece of reasoning or a judgement. All things considered, semantic abilities are those structuring abilities required for forming an unrestricted universe of discourse. A generalized pedagogy for the generation and augmentation of *sm*–abilities consists of training regimens in such structuring domains:

(a) Base Semantic Structuration

(a-1-1) Protoconceptual labelling: rudimentary classification by assigning labels/names to items—which are available to sensation—via 'reliable differential responsive disposition' (RDRD).[217] For example, the nonlinguistic $\mathbb{K}$ can be trained like a parrot to make the noise (not to be mistaken for a saying) 'That's black' in the presence of the heap of black. Here, the RDRD-performance 'That's black' in the presence of a black item imposes classification on the stimuli, thus differentiating those which would from those which would not trigger the response of the given kind by practicing that particular RDRD.

(a-1-2) Description and explanation: placing labels into a space of implications where classification is coupled with explanatory relations which can be expressed by *modal vocabulary*. An empirical description must then have both inferentially articulated circumstances for the appropriate application of labels and inferentially articulated appropriate consequences of the application of labels.

- Material Inference
  - Alethic modal vocabulary
    - Counterfactuals

217 R. Brandom, *Tales of the Mighty Dead: Historical Essays in the Metaphysics of Intentionality* (Cambridge, MA: Harvard University Press, 2002), 349–50.

  - Context-sensitivity handling (semantic consciousness of contexts and circumstance)
  - ⊙*fm*-Resource-sensitivity handling (semantic consciousness of contexts and contextual premises as logical resources)
  - Resolving conflict between different counterfactuals in one context
  - Integration or separation of different contexts
  - ⊙*fm*-Possible world representation, where the meaning or sense of an expression can be accounted for not simply by its reference in the actual world, but also by what the expression would have referred to, had the actual world been different, i.e., from the counterfactual standpoint of possible worlds that are as actual as *this* actual world of reference.[218]

— Belief revision or commitment updating

  — Non-monotonic and defeasible reasoning, i.e., a reasoning in which conclusions can be retracted based on new evidence.

  - Finding defeasors or counter-defeasors for acquiring a new belief or preserving an existing one based on the incompatibility of practical commitments/beliefs or lack thereof (cf. addition or removal of premises in the light of the relation between the control set and the context in the match example discussed above).

(a-1-3) <u>Intentional vocabulary:</u> what one uses in order to ascribe claims, beliefs, desires, or intentions that p.

(a-1-4) <u>Normative vocabulary:</u> what one uses in order to ascribe commitments or entitlements to a claim that p.

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218 See D. Lewis, *On the Plurality of Worlds* (London: Blackwell, 2001).

(a-1-5) ⊙*mf*-Non-axiomatic 'coherentist' theory formation: theories which are not axiomatic since they are not built on established truths or truth-givens, but rather are constructed out of truth-candidates whose cohering web of inferential interrelations not only decide which truth-candidates must remain, be modified, or discarded, but also make explicit the structure of theory qua system of structuration.

(b) Experimental Semantic Structuration

(b-1) Logics of discovery

— Abductive reasoning (take for instance Peirce's example of the logic of surprise: An anomaly or a surprising fact, C, is observed; But if A were true, C would be a matter of course. Hence, there is reason to suspect that A is true.[219] Here hypothesis A is suspected or conjectured to be true even though A may be false, i.e., it is tentatively believed on reasonable grounds that A is true.[220] In this framework, the observation of an anomaly and its corresponding framed hypothesis call for the revision and expansion of the theory that covered that class of observations so as to accommodate the anomalous observation. Thus, abductive reasoning can be understood as that type of reasoning that instigates a change in epistemic attitudes, cf. belief revision.)

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219 C. S. Peirce, *The Collected Papers of Charles S. Peirce* (8 vols. Cambridge, MA: Harvard University Press, 1974), vol.5, §189.

220 The role of this tentative belief can be more accurately formulated as follows:
'[It is *reasonable to believe that* the best available explanation of a fact is true.]
F is a fact. 221
Hypothesis *H* explains *F*.
No available competing hypothesis explains *F* as well as *H* does.
Therefore, it is *reasonable to believe that H* is true.' A. Musgrave, 'Popper and Hypothetico-Deductivism', in *Handbook of the History of Logic: Inductive Logic* (Amsterdam: Elsevier, 2004), 228.

- Abductive hypothesis construction or framing of conjectures (abductive 'nonpredictive' hypotheses allow for the explanation of both a proposition *and* its negation).
- Abductive model-based reasoning where models accommodate different explanations (of observed facts) and where new beliefs can be adopted and old beliefs can remain so long as they cohere (cf. coherentist theory formation and Brandom's material incompatibility and inferential consequence relations).[221]
- Model pluralism: the availability of many different explanatory schemas—weak and predictive—and their corresponding models so as to enable not only the discrimination of some explanations as preferable to others but also an increase in the range of explanation to cover new observations, anomalies, or surprising facts.

- Analogical reasoning: the exploration of the outcome of the structural alignment of the shared relational pattern between two or more contextually contiguous concepts, ideas or models. For instance, think of the Archimedean method of solving geometrical problems by inventing a mechanical analogue: e.g., a lever for solving the problem of how much bigger a cylinder is than a sphere of the same radius by articulating the relation between the weights of a cylinder solid and a sphere solid of the same radius (both made of the same material) via an adjustable lever (i.e., with a moving fulcrum) capable of balancing their weights. In this form of analogy, to solve a geometrical problem/idea, a mechanical analogue, interpretation, or metaphor of the geometrical problem is introduced. The analogical solution obtained from the machine analogue together with its constitutive

Material incompatibility and inferential consequence relations refer to 'incompatibility and inferential relations that hold in virtue of what is expressed by non-logical vocabulary. Thus claiming that Pittsburgh is west of New York City has as a material inferential consequence that New York City is east of Pittsburgh, and is materially incompatible with the claim that Pittsburgh is a prime number.' Brandom, *Reason in Philosophy*, 36.

mechanical reasoning is then mapped onto and reinterpreted as the geometrical solution and its constitutive geometrical reasoning.

— Metaphorization or conceptual cobordism:[222] how to derive a new higher-order structure from two different cognitive structures by constraining operations that allow the drawing of a contiguous contextual boundary between them through which analogical transfers and the synthesis of a third higher-order structure can be obtained. The role of metaphors in discovery can be compared, following Gilles Châtelet, to a Trojan horse that takes the cognitive habits of one context or field of thought and deploys them into another, thus setting in motion a whole dynasty of problems otherwise invisible from the perspective of any one field alone.[223]

(B) *sf*-abilities: In contrast to *sm*-abilities, *sf*-abilities can be characterized as *structure-encoding* abilities, or more generally as abilities whose main point of emphasis is on the formal or syntactic aspects of structuration. Roughly speaking, syntactic abilities or formal axiomatic abilities are required for constituting specialized domains of discourse qua sciences. They can be understood as (formal) calculi, from something like situation calculus for reasoning about dynamic domains to event calculus (representing and reasoning about events) to process calculus, proof calculus, etc. As evolved and explicitly formal structure-encoding abilities, syntactic abilities

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222 Roughly, cobordism is an equivalence relation between two manifolds of the same dimension. Two manifolds are considered equivalent if their disjoint union $\sqcup$ is the boundary (*bord*) of another manifold. A famous intuitive example of cobordism is a pair of pants. Think of the disk representing the waist as the manifold M and two disks representing the cuffs of a pair of pants as the manifold N. Their cobordism (or common boundary) can be expressed as the boundary of a higher-dimension structure (n+1-dimensional manifold W) which maps the cuffs to the waist, i.e., the boundary (a closed manifold δW) outlining the pair of pants itself. Cobordism then can be formulated as $\delta W = M \sqcup N$.

223 On the power of metaphors in the history of science particularly at the intersection of mathematics and physics, see G. Châtelet, *Figuring Space*, tr. R. Shaw and M. Zagha (Dordrecht: Kluwer, 2000).

are primarily the objects of what Robert Harper dubs the 'holy trinity of computation'—namely logic, mathematics and computer science or proofs, programs and categorical structures.[224] Just as semantics possesses a hierarchical complexity where conceptualization and the role of concepts become increasingly more involved at higher levels, so syntax also has its own hierarchical complexity. The complexity of syntactic abilities can be mapped onto two different hierarchies, pure formal grammar (à la Chomsky's hierarchy of syntax) and formal axiomatic theoretical structures (à la Stegmüller's hierarchy of axiomatics) which concerns the axiomatization of theories. The difference between these two formal hierarchies lies in their approach to syntax. Whereas formal grammar focuses on pure generative syntax and its computational-algorithmic properties, the axiomatic hierarchy deals with the different types of axioms through which different kinds of axiomatic theories (whether quasi-formal or formal) can be constructed. In this respect, formal grammar can be approximately mapped onto computational abilities (recursive pattern matching, algorithmic design, rules of pattern recognition, etc.) while the axiomatic hierarchy can (again, roughly) be mapped onto the logico-mathematical abilities required for theory construction in the domain of exact and specialized sciences.

(a) Hierarchy of formal grammar as the domain of basic formalization abilities: In terms of pure syntax, syntactic complexity consists of the (recursive) processes required for generating syntactic languages or encoding structures, formal grammatical properties that specify levels of encoding or formal languages, and the automata necessary for computing them. In this hierarchy, computational power and complexity, and sophistication of encoding, increase from lower levels of syntax to higher levels. In tandem with the increase in computational capacities (computational cost), the demand for memory resources also increases.

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224 For a brief introduction to computational trinitarianism see R. Harper, *The Holy Trinity* (2011), <https://existentialtype.wordpress.com/2011/03/27/the-holy-trinity/>.

Consequently, with the increase in computational costs and resources from the bottom to the top, *effective computability* decreases.[225]

(b) Hierarchy of axiomatics as the domain of abilities (of logic and mathematics and computation) required for the construction of formal theories as employed in specialized sciences: As formal *axiomatics*—that is, systems required for forming specialized axiomatic theoretical structures—the complexity of the formal can be elaborated as the hierarchy of axiomatics and the different types of formal theory-structures afforded by different classes of axiomatic systems. In *The Structure and Dynamics of Theories*, Stegmüller classifies axiomatic systems (or calculi) into five forms of axiomatization, with each form having the capacity to construct a distinct class of structuration qua formal axiomatic theory:[226] (1) intuitive axiomatization (axioms as self-evident *truth-sentences*) as in Euclid's *Elements*; (2) informal Hilbertian (set-theoretic) axiomatics or abstract qua nonintuitive axiomatics where axioms are *sentence-forms* belonging to the ordinary language of discourse; (3) formal Hilbertian axiomatics (axioms as formulas and axiomatizations as *calculi of formulas*) comprising tuples $\langle S,A,R \rangle$ where S is a syntactic system, R inference rules for deriving formulas from formulas, and A a subclass of axioms belonging to the axiomatic system based on the construction of a completely formal language; (4) informal (naïve) set-theoretical axiomatization, where axiomatization is based on the definition of a set-theoretical predicate and axioms are elements of an introduced *set-theoretic predicate.* It is called informal axiomatization since set-theoretic predicates are introduced at the ordinary and intuitive level of discourse rather than in the framework of the formal system of set theory itself; (5) explicit predicate or explicit concept for an axiom system, which is the formal equivalent of informal naïve set-theoretic

225 See M. Li and P. Vitányi, *An Introduction to Kolmogorov Complexity and Its Applications* (Dordrecht: Springer, 2008), and A. Minai, D. Braha, and Y. Bar-Yam, *Unifying Themes in Complex Systems* (Dordrecht: Springer, 2010).

226 Stegmüller, *The Structure and Dynamics of Theories*, 30–37.

> axiomatization. Here axioms—in comparison and contradistinction with the fourth axiomatic system—are *explicit predicates belonging to the formal system of set theory*. In the case of each of these calculi, by 'assigning to the individual terms in the axioms definite objects and to the property and relation predicates properties and relations, one obtains an interpretation of the axiom system'.[227]
>
> From the perspective of constructing models, the hierarchy of axiomatization or calculization of theories is intrinsically connected with the semantic dimension, since the concept of formal model is based on the conversion of the syntactically defined formal language—via the introduction of an interpretation—into a semantic system where the concept of validity as relating to terms, statements, and applications of the model to the data under consideration can be made precise. Without this conversion, the objectivity of a model cannot be sufficiently established.

Given the importance of the pure formal grammatical and axiomatic aspects of syntax for computational and theoretical abilities, *sf*-abilities are absolutely necessary for the encoding and construction of formal and specialized fields of structuration—that is, for forming complex *models* of the world.

The goal of the catalogue above is to show not only that we can think about the cultivation of our child AGI in terms of a combinatorial calculus of structuring powers of the mind, where we can map one ability to another or decompose a complex ability to simpler ones, but also that such a curriculum requires a diverse range of educational methods. As Brandom suggests, the problem of generalized pedagogy is the central problem of artificial general intelligence. The graduation from a CHILD to an intelligence that encounters itself in an objective world and thus is capable of reimagining itself in accordance with an expansive field of intelligibility requires a back-and-forth movement between the trainee ($\mathbb{K}$) and the trainers ($\mathbb{S}$ and $\mathbb{M}$). Such a movement is built on a pedagogical

227 Ibid., 32.